Predicting the Supersonic Aerodynamics of Very-Low-Aspect-Ratio Lifting Surfaces

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Improved methods for predicting aerodynamics of lifting surfaces are needed at the very low aspect ratios found in wing (housing) designs of span limited missiles and at moderate supersonic to hypersonic Mach numbers. A successful empirical method for estimating the normal force coefficient, C_N, and center of pressure location, X_{cp}, for this class of surfaces has been developed for unbanked ($\phi = 0$ deg) cruciform wing configurations and for wing configurations banked at $\phi = 45$ deg. The values of C_N and X_{cp} obtained using these empirical curves provide good estimates at Mach numbers above 2.5, and for angles-of-attack to 20 deg for thick and thin surfaces mounted in cruciform on cylindrical bodies.

Nomenclature

AR = aspect ratio $= b^2/S_W$
$b/2$ = exposed semi-span of a lifting surface mounted on a body of revolution, in.
C_N = normal force coefficient, normal force (lb)$/qS$ resulting from external aerodynamic forces
ΔC_{N_W} $= C_{N_{BW}} - C_{N_B}$ (excludes internal forces if originally present, as in ducts)
C'_{N_W} = normal force coefficient of wing (or housing) normalized to wing (housing) planform area and with carryover removed, $\Delta C_{N_W}(S/S_W)/(K_W + K_B)$
ΔC_{N_ϕ} $= \Delta C_{N_W}$ at $\phi = 45$ deg $- \Delta C_{N_W}$ at $\phi = 0$ deg
C_r = root chord, in.
d = reference length; diameter of body on which lifting surfaces are mounted, in.
K_B, K_W = Morikawa's interference factors[11]
ℓ_a = length of cylindrical body aft of the trailing edge of the wing (housing), in.
M = Mach number
q = dynamic pressure, lbs/in.^{2}
S = reference area, $\pi d^2/4$, in.^{2}
S_W = planform area of housings (wings) at $\phi = 0$ deg, in.^{2}
S_P = planform area of housings (wings) at $\phi = 0$ deg plus planform area of body section aft of the leading edge station at the root, X_{LE}, to the base of the body in.^{2}
t = average thickness of lifting surface, in.
X = body station; $X = 0$ at nose tip of body, in.,
X_{cp} = center of pressure location, in.
X_{LE} = location along the X axis of wing (housing) leading edge at the root, in.
$\Delta \bar{X}$ = location of X-centroid of wing planform area measured from X_{LE}, in.
ΔX $= X - X_{LE}$, in.
α = angle of attack; angle between the velocity vector and the longitudinal axis of the body, deg
β $= \sqrt{M^2 - 1}$
Λ = leading-edge sweep angle for delta wings, deg
ϕ = aerodynamic roll angle; at $\phi = 0$ deg, two of the cruciform lifting surfaces are in the plane of α, deg.

Subscripts and Terminology

B = body alone
BW = body-wing or body-housing combination
W = wing or housing (E_1 is housing with duct)

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Introduction

LIFTING surfaces of very low aspect ratio are incorporated in the design of some missiles because of stowage requirements for compactness. These lifting surfaces may be used to house electronics or other equipment or serve as ducts, as in the case of side-mounted inlets on proposed ramjet missiles.

Empirical estimates of the normal force coefficient C_N and center-of-pressure location X_{cp} for these surfaces are difficult because the shapes are usually unique for each new missile design (e.g., Fig. 1), and therefore the limited test data available are invariably for shapes that are quite different from a proposed shape. Existing empirical methods[1] have been derived for a specific class of surfaces and apply to the lower end of the range of Mach numbers of interest in this study.

Simple theoretical methods that have been used with limited success do not take into account the effects of Mach number as Fig. 2 shows. Those methods that do consider Mach number effects do not apply at the very low values of aspect ratio inherent to these types of surfaces. These concerns have been expressed for some time.[2,3] A need exists, therefore, for either a test data base for a general class of lifting surfaces with low aspect ratio or for the development of simple predictive methods using the available data. The objective of this investigation was to develop empirical methodology, using existing data, for preliminary estimates of C_N and X_{cp} of lifting surfaces with very low aspect ratio operating at $M \geq 2.5$, at moderate angles of attack, and configured in cruciform on a missile body.

Methodology

The approach taken was to develop empirical correlations of C_N and X_{cp} using existing component data from configurations consisting of lifting surfaces of very low aspect ratio mounted in cruciform on bodies of revolution. The test data come from a variety of wing (housing) configurations. Altogether, test data from 27 combinations of Mach numbers and configurations were used to generate the empirical curves

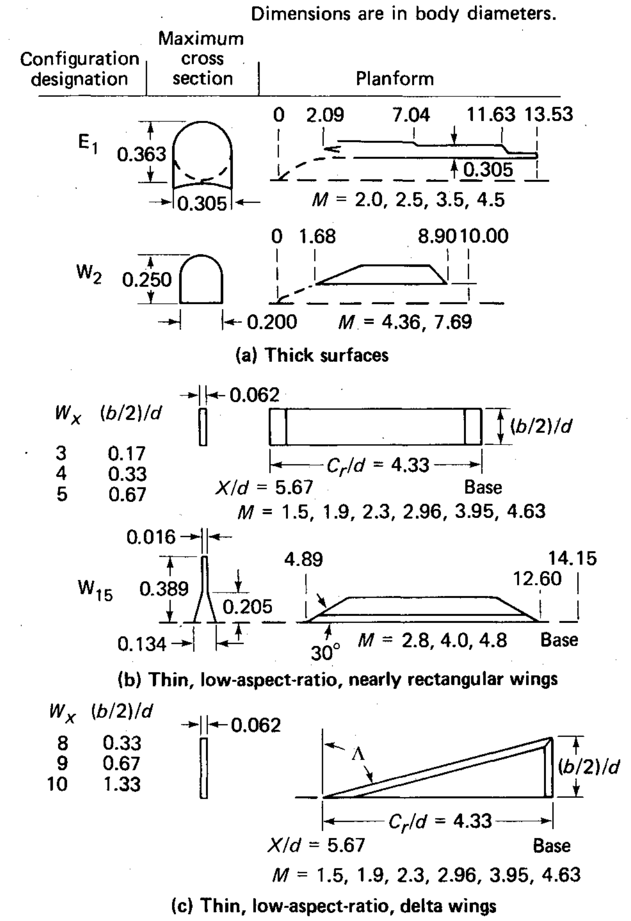

Fig. 1 Sample of wing (housing) configurations used in analysis.

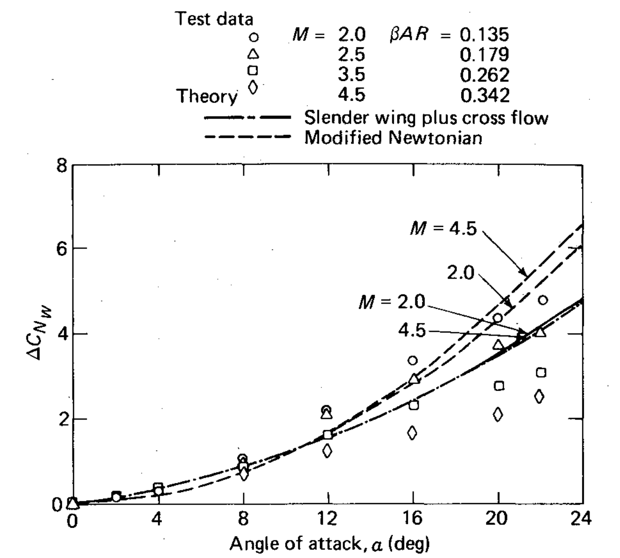

Fig. 2 Comparison of C_N data from E_I housing with two simple predictive methods, $\phi = 0$ deg.

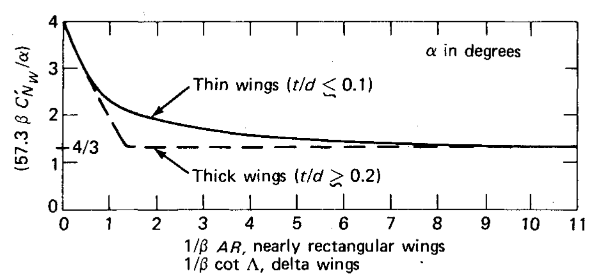

Fig. 3 Proposed design charts for $\beta C_N'/\alpha$ of low Æ wings, $M \gtrsim 2.5$, $\phi = 0$ deg.

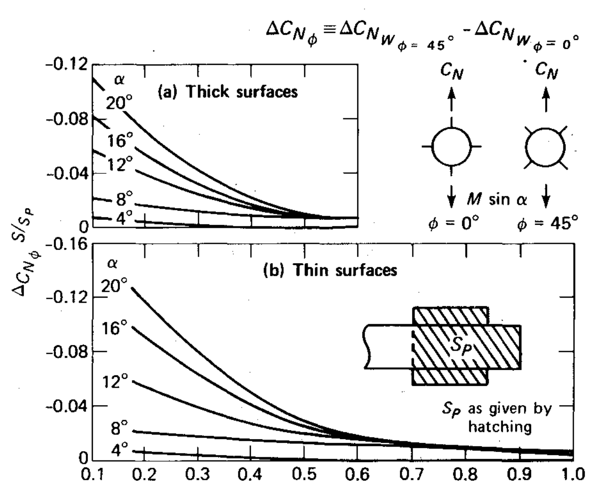

Fig. 4 Empirical charts for estimating C_N at $\phi = 45$ deg of very-low-aspect-ratio, cruciform, nearly rectangular lifting surfaces.

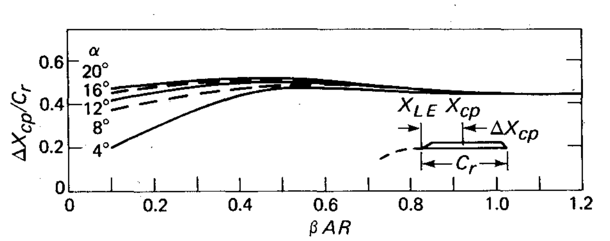

Fig. 5 Empirical charts for $\Delta X_{cp}/C_r$ of nearly rectangular, low Æ lifting surfaces (cruciform configurations), $\phi = 0$ deg, thick and thin surfaces.

proposed herein. These cases and 29 additional combinations were used to evaluate the application of the empirical methods. These sources represent 20 different wing and housing configurations. The primary sources of test data for the development of the method presented herein were various Johns Hopkins University Applied Physics Laboratory (JHU/APL) aerodynamic research programs and NASA data[4] on rectangular and delta wings.

Successful correlations of C_N and X_{cp} were obtained with β Æ for wings whose planforms are nearly rectangular and with $\beta \cot \Lambda$ for delta wings. Empirical curves, which are summaries of these correlations, constitute the proposed methodology for determining C_N and X_{cp} of lifting surfaces of very low aspect ratio (Figs. 3 through 7).

At $\phi = 0$ deg, $\beta C_{N_W}'$ was found to correlate with β Æ (or $\beta \cot \Lambda$) when linearized with α over the test range of α as summarized in Fig. 3. Different correlation curves are obtained for thick wings (defined herein as $(t/d \gtrsim 0.2)$ and thin wings $(t/d \lesssim 0.1)$. A linear interpolation is suggested for intermediate thicknesses.

An example of the data correlation for C_{N_W}'/α at $\phi = 0$ deg is given in Fig. 8. Also shown are the values of $\beta C_{N_W}'/\alpha$ predicted from linear and slender wing theories[5] for rectangular wings. It is seen that the difference between experiment and theory (given by these simple methods) is very large for β Æ $\gtrsim 0.67$, i.e., $(1/\beta$ Æ$) \approx 1.5$. Note that the theoretical values of C_{N_α} are lift curve slopes at $\alpha = 0$ deg, whereas the test values are the mean values of C_N'/α obtained from the full range of α tested. For the test cases where C_N was nearly linear with α $(M \gtrsim 3.0)$, these two values should be about the same. These theoretical methods are usually recommended in various handbooks and textbooks because of their success in predicting C_{N_W}'/α at low values of α. Their success has been

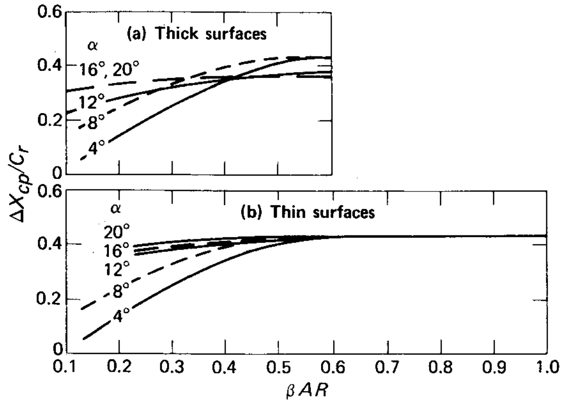

Fig. 6 Empirical charts for $\Delta X_{cp}/C_r$ of nearly rectangular, low AR, lifting surfaces (cruciform configurations), $\phi = 45$ deg.

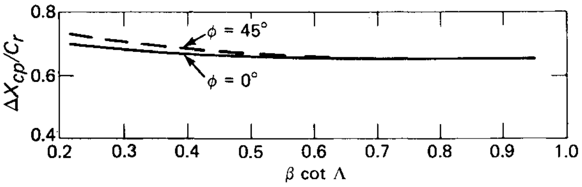

Fig. 7 Empirical chart for estimating X_{cp}/C_r for thin delta wings (cruciform configurations), 4 deg ≤ α ≤ 20 deg.

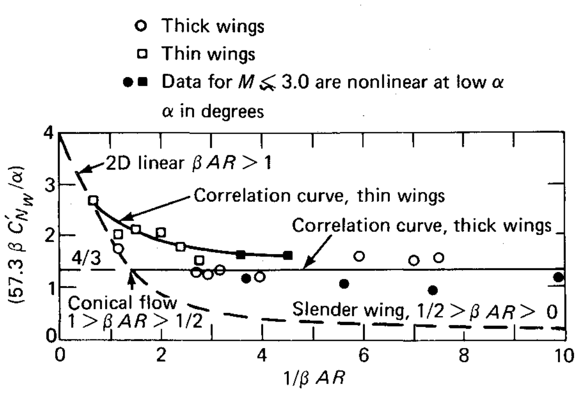

Fig. 8 Correlation of test data on C_N of nearly rectangular wings, $\phi = 0$ deg.

demonstrated by several investigators at low values of α and at low supersonic Mach numbers. The inadequacy of these theoretical methods for predicting ΔC_{N_W} without adding a nonlinear term such as cross-flow lift was demonstrated by Flax and Lawrence[2] in 1951. Cross-flow lift for wings is a concept taken from cross-flow lift on cylinders that attempts to account for the vortex lift. The cross-flow drag value used in determining cross-flow lift is basically an experimental value obtained for a limited class of wings.[2,6] More recent approaches use the concept of leading-edge and side-edge suction[7,8] to account for nonlinear lift. As far as can be established from the literature, this approach is not applicable to the wing geometries of interest in this study.

Similar comparisons of theory with test data from delta wings show disagreement with linear theory. Specifically, the test values of $\beta C'_{N_W}/\alpha$ for delta wings do not tend to 4 at $\beta \cot \Lambda = 1$ as predicted by linear theory but rather at $\beta \cot \Lambda = \infty$, which is synonymous with predictions for rectangular wings.

The empirical curves for estimating ΔC_{N_W} at $\phi = 45$ deg (Fig. 4) were generated from correlations of the incremental test value

$$\Delta C_{N_\phi} \equiv \Delta C_{N_{\phi=45}} - \Delta C_{N_{\phi=0}}$$

It was expected that this difference in C_N would result mainly from the difference in the viscous, or cross-flow, lift contribution from the banked wings and from the difference in mutual body-wing interference. Indeed, ΔC_{N_ϕ} was found to correlate with βAR when it is referenced to the planform area of the wings and the planform area of the body region influenced by the wings. With this planform area simplified as shown in the sketch in Fig. 4 as S_P, the increment, ΔC_{N_ϕ}, was found to correlate with βAR when normalized by the area ratio S/S_P. Note that ΔC_{N_ϕ} includes the difference in mutual wing-body interference between the $\phi = 45$ deg and $\phi = 0$ deg cases. Different correlations were found for the thick ($t/d \gtrsim 0.2$) and thin ($t/d \lesssim 0.1$) nearly rectangular surfaces.

For the thin delta surfaces,

$$\Delta C_{N_\phi} \approx 0$$

The center-of-pressure location of the wings (housings) measured from the wing leading edge, when normalized by the root chord, C_r, was also found to correlate with βAR for the rectangular wings and with $\beta \cot \Lambda$ for the delta wings (Figs. 5, 6, and 7). At $\phi = 0$ deg, the correlation curves of X_{cp} were found to be essentially the same for the thick and the thin nearly rectangular wings (Fig. 5); as expected they are different for rectangular and delta wings (Figs. 5 and 7).

At $\phi = 45$ deg, separate correlation curves of the center of pressure, $\Delta X_{cp}/C_r$, were found for thick and thin nearly rectangular surfaces (Fig. 6). For the delta wings, $\Delta X_{cp}/C_r$ was found to be nearly independent of ϕ and α for the range of $\alpha = 4$ to 20 deg (Fig. 7).

An example showing the data correlation for $\Delta X_{cp}/C_r$ is shown in Fig. 9 for $\phi = 0$ deg and three selected values of α.

Limits of Application

The empirical curves of C_N and X_{cp} given in Figs. 3 through 7 are derived herein from the correlations of test data from a wide variety of wing-body geometries (see Fig. 1) tested over a wide range of Mach numbers, and therefore can be expected to have wide application. Some limits on the application of these curves have been deduced in generating

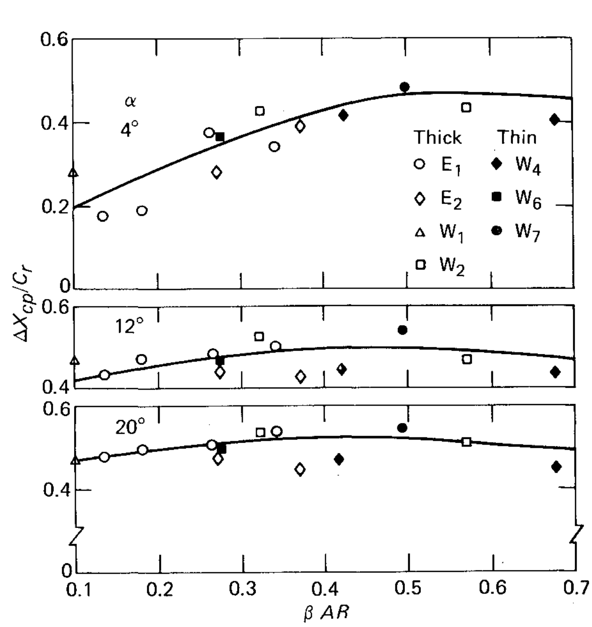

Fig. 9 Sample correlation of test data on $\Delta X_{cp}/C_r$ for nearly rectangular wings in cruciform, $\phi = 0$ deg.

Table 1 Region of validity ($\alpha > 0$ deg for all parameters)

			$(b/2)/d$	
Parameter	M	ℓ_a/d	Rectangular wings	Delta wings
$\beta C'_{N_W}$ at $\phi = 0$ deg	$2.5 \le M \le 8$	< 5	0.17 to 0.67	0.09 to 1.33
$\Delta C_{N_\phi} S/S_P$	$2.5 \le M \le 8$	< 2	0.21 to 0.67	0.21 to 1.33
$\Delta X_{cp}/C_r$	$2.5 \le M \le 8$	< 2	0.21[a] to 0.67	0.21[a] to 1.33

[a]For $\Delta X_{cp}/C_r$, span is nearly constant along the chord.

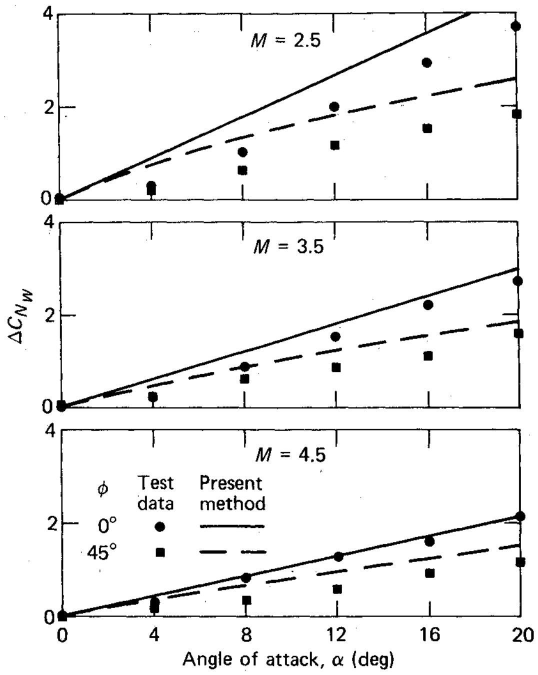

Fig. 10 Comparison of predictions and test data, ΔC_{N_W} of E_I housing.

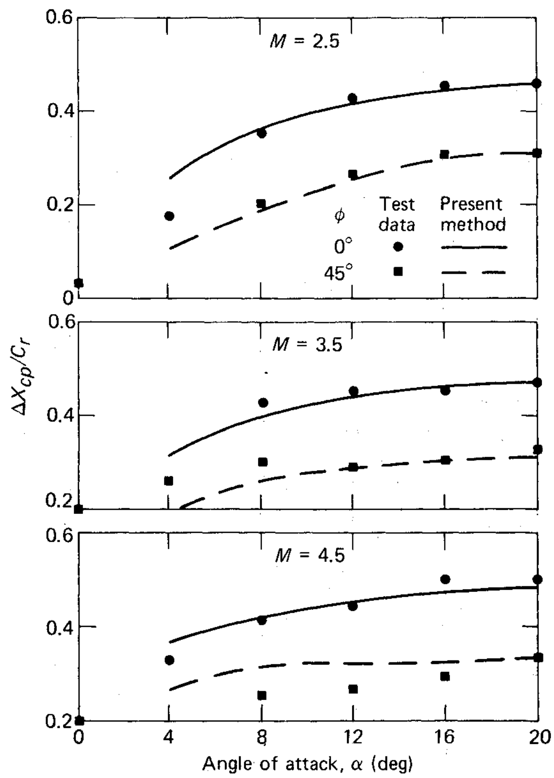

Fig. 11 Comparison of predictions and test data, ΔX_{cp} of E_I housing.

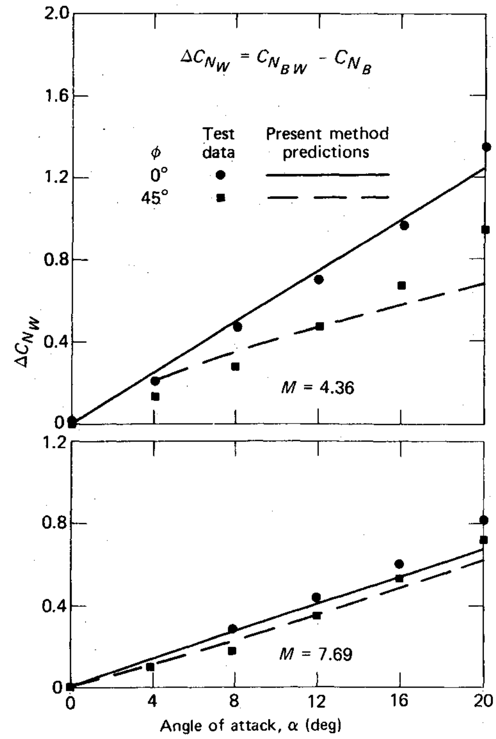

Fig. 12 Comparison of predictions and test data, ΔC_{N_W} of W_2 wings.

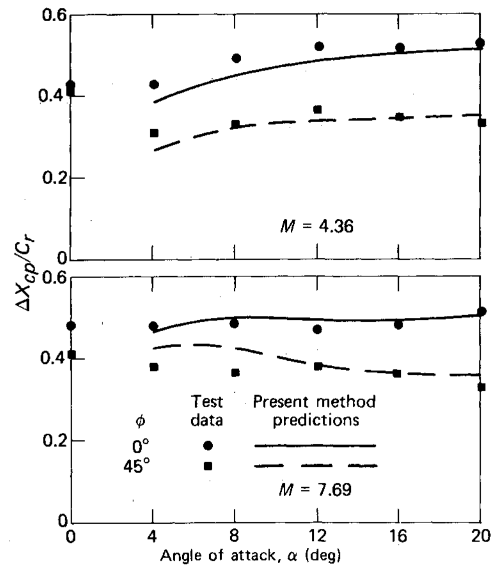

Fig. 13 Comparison of predictions and test data, ΔX_{cp} of W_2 wings.

the correlation curves and in comparing the calculated values of C_N and X_{cp} with test data. These limits are summarized in Table 1.

The Mach number limitation is imposed because the values of ΔC_{N_W} calculated from the empirical curves for some cases do not compare well with test data at $M \le 2.5$.

The limits given above on aft-body length are suggested by both the present data and by limited test data[9] for aft-body lengths of 1.1, 3.1, and 5.1 diameters.

The lower limit in span on the correlation of $\Delta C_{N_\phi} S/S_P$ (viz. 0.21) is imposed because the $\Delta C_{N_\phi} S/S_P$ data for these smaller wings did not correlate with the data for the larger wings. These smaller wings differ in crossflow from the others in that, at $\phi = 45$ deg, the diameter of the body is greater than the projected span of the wing-body combination. It is ex-

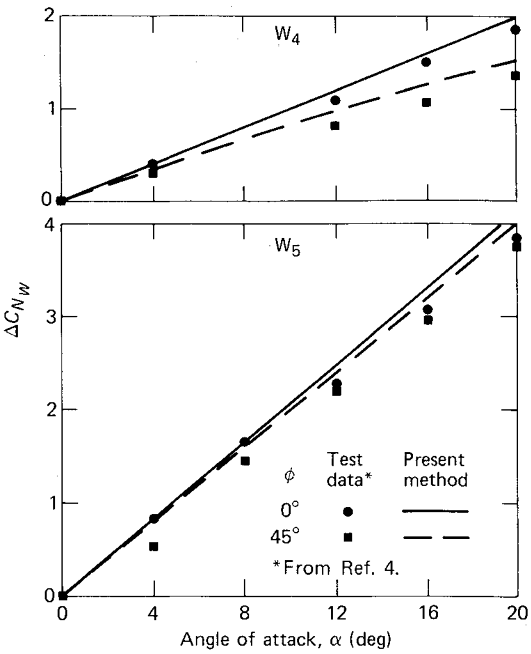

Fig. 14 Comparison of predictions and test data, ΔC_{N_W} of thin rectangular wings, $M = 2.96$.

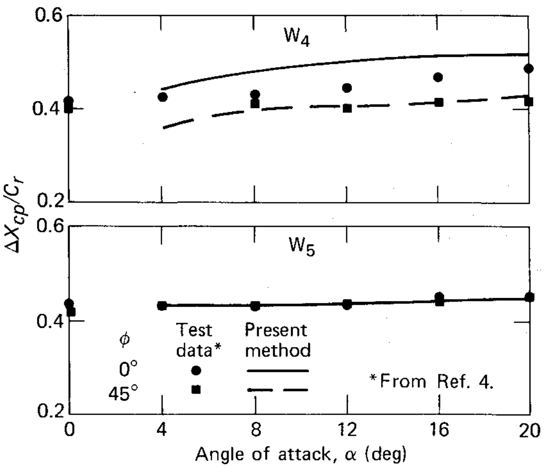

Fig. 15 Comparison of predictions and test data, ΔX_{cp} of thin rectangular wings, $M = 2.96$.

pected that for these smaller wings the wing forces are dominated by the body crossflow, and thus the effect of ϕ on $C_{N_{BW}}$ is less for the smaller wings. An analogous effect of wing span on wing-wing interference as a function of roll orientation is shown by Nielsen.[10] Restated, Nielsen's results show that wing-wing interference is not influenced by an adjacent wing when $(b/2)/d \lesssim 0.2$.

The other limits given for span simply reflect the range of the test data used in the correlations for the rectangular and delta wings. A limitation stated in Table 1 for estimating $\Delta X_{cp}/C_r$ is that the span of the surface should be nearly constant. Test data from two surfaces that did not have constant span were used in the correlations of ΔX_{cp} in this study. It was found that when ΔX_{cp} for these surfaces is normalized by $2\Delta \bar{X}$ instead of C_r, the test data fall within the spread of the rest of the data. The applicability of the present method to wings having sections of different spans, however, requires more verification. The condition that $\alpha > 0$ deg is imposed because the procedure in deriving the correlations of C'_{N_W} at $\phi = 0$ deg involved linearization of the data.

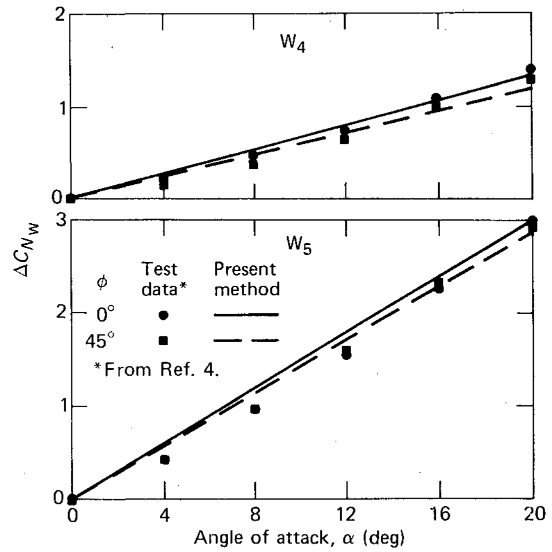

Fig. 16 Comparison of predictions and test data, ΔC_{N_W} of thin rectangular wings, $M = 4.63$.

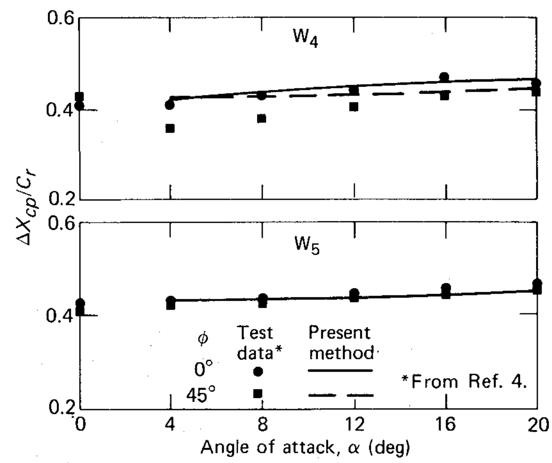

Fig. 17 Comparison of predictions and test data, ΔX_{cp} of thin rectangular wings, $M = 4.63$.

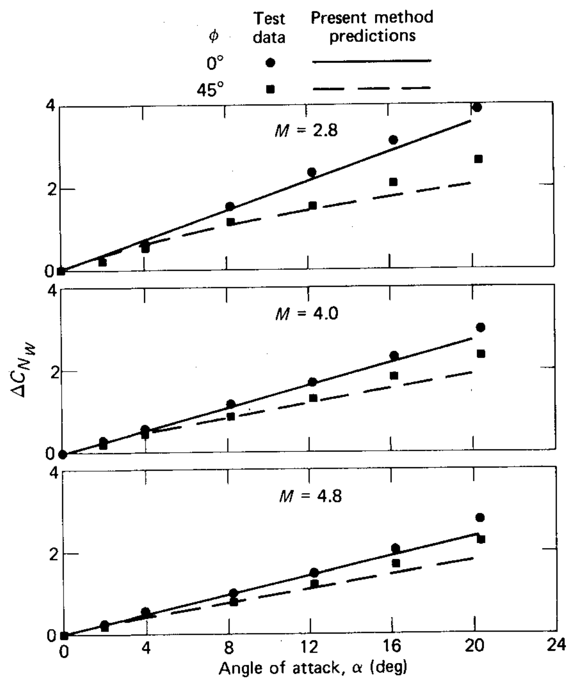

Fig. 18 Comparison of predictions and test data, ΔC_{N_W} of W_{15} wings.

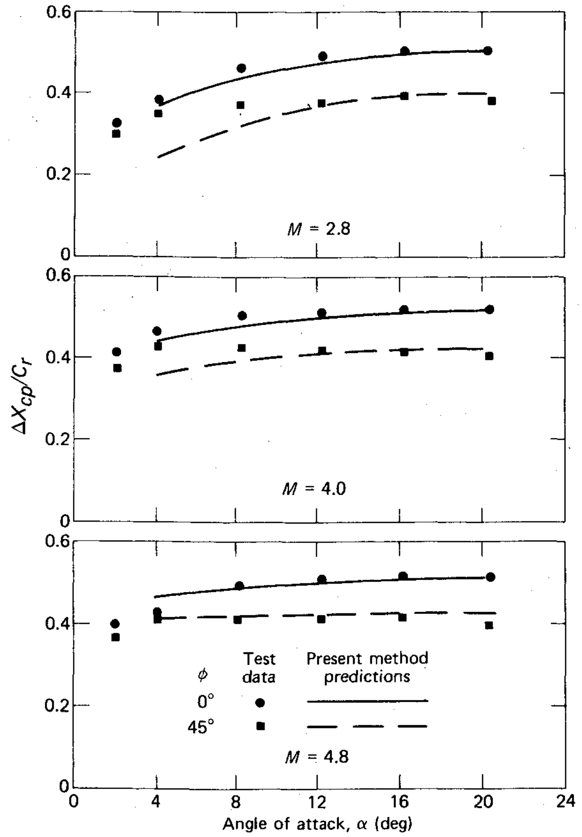

Fig. 19 Comparison of predictions and test data, ΔX_{cp} of W_{15} wings.

Summary of Procedure for Using Empirical Design Curves

The procedure given here for estimating C_N and X_{cp} of low-aspect-ratio lifting surfaces assumes that the estimates are to be made for surfaces mounted in cruciform (at either $\phi = 0$ or 45 deg) on a cylindrical body. The body length aft of the wing (housing) trailing edge is assumed to be less than two body diameters. The estimates of C_N and X_{cp} for the body alone are not required to extract C_N and X_{cp} for the low-aspect-ratio surfaces, but the assumed geometry of the body of revolution contributing to S_P will be required to derive both the carryover factors and the incremental value of ΔC_{N_W} at $\phi = 45$ deg.

Normal Force Coefficient C_N at $\phi = 0$ Deg

$$C_N = \left(57.3\beta C'_{N_W}/\alpha\right)\frac{\alpha}{57.3}\frac{(K_W + K_B)}{\beta}\frac{S_W}{S}$$

where $(57.3\beta C'_{N_W}/\alpha) = f(\beta \mathcal{R}$ or $\beta \cot \Lambda)$ is given in Fig. 3, with α given in degrees. The factors $(K_W + K_B)$ are the Morikawa carryover factors from Ref. 11 used in the formulation of the empirical curves.

Normal Force Coefficient C_N at $\phi = 45$ deg

Nearly Rectangular Surfaces

$$C_{N_{\phi=45^\circ}} = C_{N_{\phi=0^\circ}} + \left(\Delta C_{N_\phi} S/S_P\right) S_P/S$$

where $\Delta C_{N_\phi} S/S_P = f(\beta \mathcal{R}, \alpha)$ is given in Fig. 4. S_P is the projected planform area of the wings (at $\phi = 0$ deg) plus the body section (upon which the wings are mounted) extending from the wing leading-edge station at the root to the base of the body as shown in the sketch on Fig. 4. The value of $C_{N_{\phi=0^\circ}}$ can be either a measured or calculated value.

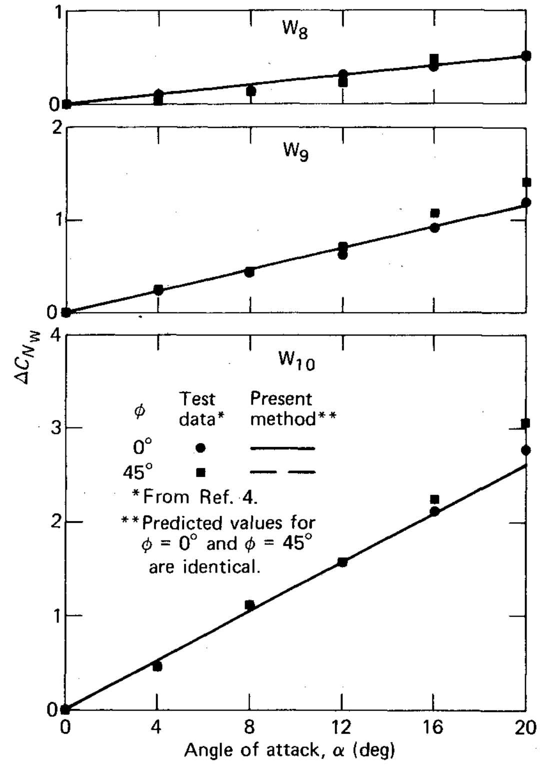

Fig. 20 Comparison of predictions and test data, ΔC_{N_W} of thin delta wings, $M = 4.63$.

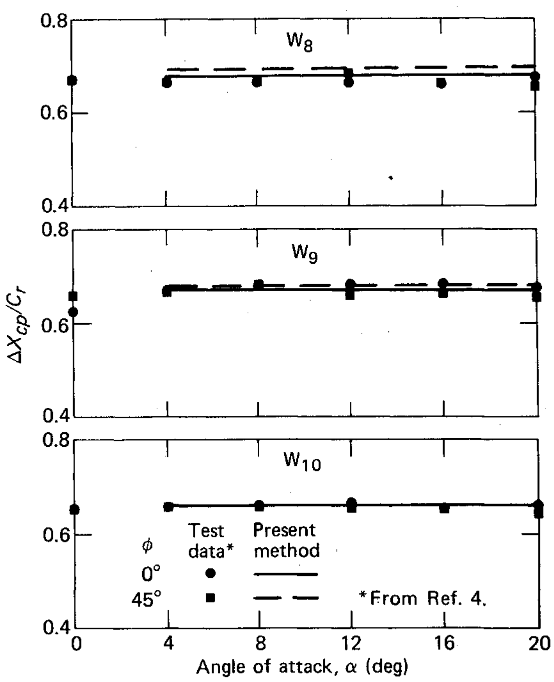

Fig. 21 Comparison of predictions and test data, ΔX_{cp} of thin delta wings, $M = 4.63$.

Delta Wings

$$\Delta C_{N_\phi} S/S_P \approx 0$$

Center of Pressure, $\Delta X_{cp}/C_r$

Nearly Rectangular Wings

The center of pressure, $\Delta X_{cp}/C_r$, is given in Figs. 5 and 6 at $\alpha \gtrsim 4$ deg.

Delta Wings

$\Delta X_{cp}/C_r$ is obtained from Fig. 7.

Comparisons of Test Values With Present Empirical Predictions

The estimates of $\Delta C_{N_W} = (C_{N_{BW}} - C_{N_B})$ and of the center of pressure $\Delta X_{cp}/C_r$ of these forces, generated herein from the empirical curves of Figs. 3 through 7, are compared in Figs. 10 through 21 with test data from the sample configurations shown in Fig. 1. Altogether, test data from 56 Mach number/configurational combinations were used to evaluate the application of the empirical curves proposed herein for estimating ΔC_{N_W} and $\Delta X_{cp}/C_r$ of wings (housings) of very low-aspect ratio. Test data from 27 of these combinations were used to generate the empirical curves. These sources represent 20 different wing and housing configurations.

With few exceptions, and with the limits discussed in the section on limits of application, the estimated values of ΔC_{N_W} and $\Delta X_{cp}/C_r$ obtained from the empirical curves for the 56 Mach number/configurational combinations provide a good prediction of the test results at $M \gtrsim 2.5$ and α to 20 deg. The poor agreement in C_N at $\alpha \approx 0$ deg is expected, because in formulating the correlations at $\phi = 0$ deg, the test data were linearized with α over the test range in α with emphasis on representing the moderate to high range in α by means of the linearization. At $M \lesssim 2.5$, Hart's empirical curves of ΔC_{N_W}[1] provide a better estimate of C_N at $\phi = 0$ deg for thin, nearly rectangular wings.

Conclusions

Empirical curves have been derived which provide good estimates at $M \gtrsim 2.5$ and α to 20 deg of C_N and X_{cp} of very low-aspect-ratio lifting surfaces mounted in cruciform at $\phi = 0$ or 45 deg on bodies of revolution having cylindrical bodies following the nose.

Acknowledgment

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